Examiner's commentary

The student's personal interest in and enjoyment of the topic is clear from the outset. The research question is a little broad and should have been refined slightly. As a result, the essay is too long and extra words in the appendix taking the essay over 4,000 words is not allowed. This essay would probably have scored higher if the extension "expanding the rules of the game" had been omitted or simplified. The extension is just more of the same techniques so doesn't add to the essay mathematically, but it takes from the essay marks for the over broad research question and for presentation. As it is, the essay is very well written, with excellent research, clear understanding and the evaluation sums up a well-argued essay. It is certainly worthy of a grade A. It is noticeable that the mathematics employed would be well within the grasp of an SL Mathematics student.

Combinatorial Game Theory in Chopsticks

Research Question:

To what extent can algorithms be used to win the finger game Chopsticks and a variation thereof involving remainders and transfers?

IBDP Extended Essay Mathematics

Word count: 3,996

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I. **Introduction**:

Chopsticks is a game with perfect information; meaning that there is no luck nor chance involved (Sakura's Zashi. 2008.). The game is simple and deterministic, which led me to being interested in solving the game. Chopsticks is a game which I often play with my friends, and although I have a general idea on how to win or at least on how not to lose, it is mostly memorization and I do not know the theory behind how the game works or why my moves work. It intrigues me to explore this game and fully understand it, such that when I'm playing I know why I'm making the moves I am. Furthermore, solving this game will hopefully introduce me to and help me to understand combinatorial game theory, a topic that I've never covered before.

My research question is "To what extent can algorithms be used to win the finger game Chopsticks and a variation thereof involving remainders and transfers?". This research question will not only guide me to finding the winning strategy to the game of chopsticks, but it will set me on a path to fully explore why and to what extent these winning strategies work. This is due to the involvement of using algorithms to win the finger game, which will require full justification and reasoning using logic and mathematics. Furthermore, besides the standard game of chopsticks, I will also be exploring the more complicated variation of chopsticks was has the two additional rules called 'remainders' and 'transfers' to further explore the game's concepts.

The base game of chopsticks has been looked into and already solved, where methods to win already exist (Revolvy. 2018). However, there is no evidence, reasoning nor algorithm supporting any of these methods, which the initial part of my research will look into. However, the expanded version of chopsticks with remainders and transfers does not have a strategy to win online. At most, I found the rules on these expanded versions, though a winning strategy and/or analysis are nowhere to be found. Thus, I will have to fully independently conduct the investigation with my own methodology. This includes making my own notation, making my own terminology, and deciding on my own which strategies to use to analyze the game.

II. The Rules of the Game:

The game of chopsticks is between two people. Each person holds out their two hands, with one stretched finger on each of them. Person A then decides to use one of his hands to 'attack' one of Person B's hands. Person B's attacked hand then increases to the sum of the two hands involved in the attack by stretching out his fingers to accommodate that amount. For example, as seen in Figure 1, Player B has 3 and 2 outstretched fingers on his hands and person A has 1 and 2 outstretched fingers on his hand, as seen in the first image of figure 1. If person B attacks person A's hand of 2 fingers with his own hand of 2 fingers, person A's receiving hand will increase to 4 fingers, which are shown in the second and third panel of figure 1 respectively. If person B instead attacked person A's hand of 2 fingers with his hand of 3 fingers, person A's hand would be completely outstretched. This hand is then out of the game and can't attack nor be attacked anymore. The goal of the game is to make your opponent stretch out all of his fingers. (Sakura's Zashi. 2008.)

Figure 1: Player A(left) vs. Player B(right) showing an initial state, an action, and the resulting state.

III. Notation Creation

According to general game theory, game notation can be categorized into extended form and normal form. Extended form involves describing a game through a game tree ("Game Theory I: Extensive Form."), while normal form involves describing a game through alpha numerals in a table form, often through a matrix. To describe a game of chopsticks, I will use the normal form since a game tree for a single run through would be inappropriate. Before brainstorming the notation, I will come up with some requirements for the notation.

Requirements:

- Able to describe the state of the game at any stage
- Able to describe the actions between states
- As compact as possible
- As complete as possible

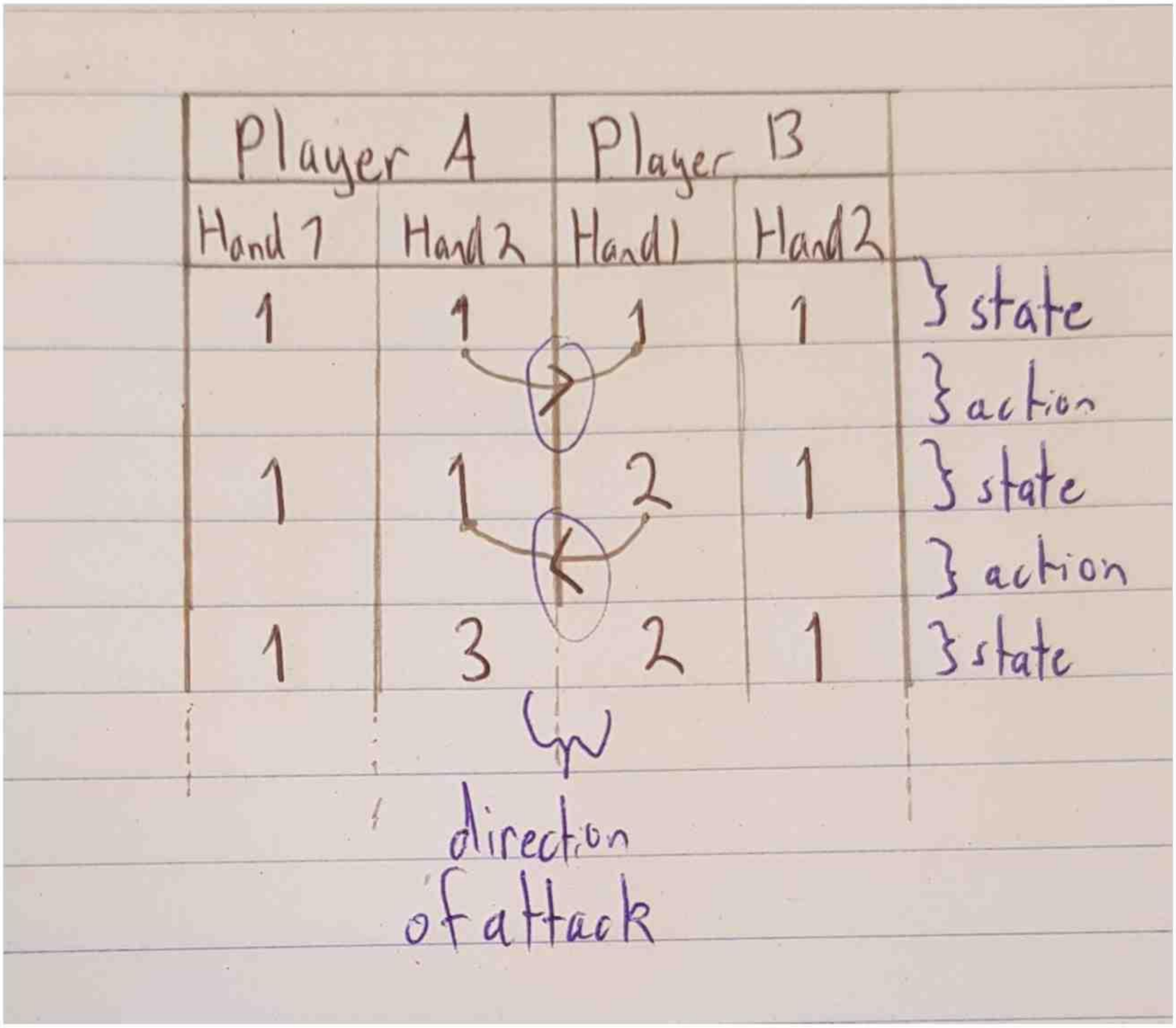


Figure 2: First notation

The first notation shown above was focused on being as complete as possible. The notation is a table, with a column for each player, and every new state displayed in the following row. Though the table includes all the needed information, the table is far from being compact and accessible, especially considering the clumsy arrows inside the table. The first notation is in the right direction, but the next objective is the optimizing and compacting of the notation.

After continuously compacting and optimizing the notation from figure 2, I reached the final notation shown in figure 3. I also tried but failed to write notations in terms of expressions and matrices. The compacting process and the attempts at notation can be found in the appendix.

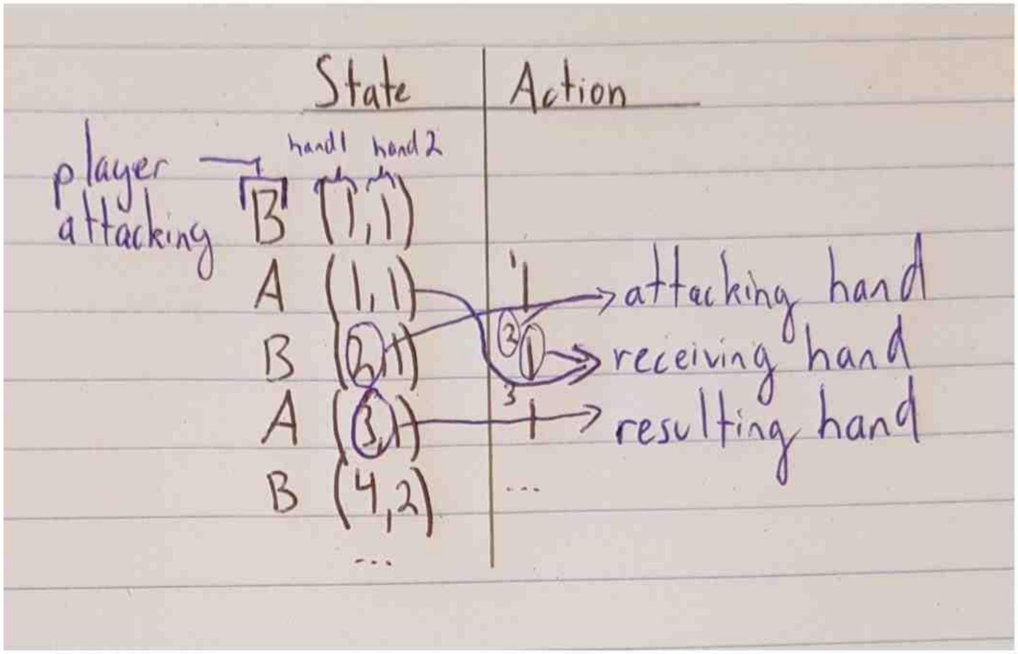


Figure 3: Final notation for Chopsticks

The final notation in figure 3 above splits the information in two halves, the state and the action. On the left side would be the state, describing the current situation, while on the same row on the right is the action, describing the change to the state in the next row. The action column consists of 2 symbols; the attacking value as superscript and the receiving value as the base.

The advantages of this notation is that it is expandable, compact, and fully describes all the relevant information about the game. However, its drawbacks include that it is difficult to understand. This is an important drawback, though this notation which aligns all the states and the actions is the closest I could get to an accessible notation.

IV. Deciding on Analysis Process

Chopsticks is a game without an element of randomness or chance, where the players have complete knowledge on the state of the game at all times. As such, Chopsticks belongs to combinatorial game theory. Combinatorial game theory studies [redacted]. This provides me with multiple methods of analyzing the game Chopsticks in order to solve it.

The first method is backward chaining which involves starting at the outcome, which in our case would be a player winning, and working your way backwards (Chein, Michel). However, I do not think this method will work since it assumes a decision from the opposite player who has the option to decide on something else, averting from the desired outcome. The second method is to start plotting out all the paths of the game. Although this can be done using my notation for many different game pathways, it is much more accessible to plot out the game using the extended form of the game, also known as a game tree. The game tree consists of nodes and edges, where each node is a point in the game where the player makes a decision, splitting up into multiple edges which are the outcome of the decision.

V. Preliminary Analysis of Basic Chopsticks

The game of Chopsticks is split up between distinct states and actions. Thus, the game can be interpreted with a game tree showing all possibilities. Figure 4 below shows the initial few steps of the game tree.

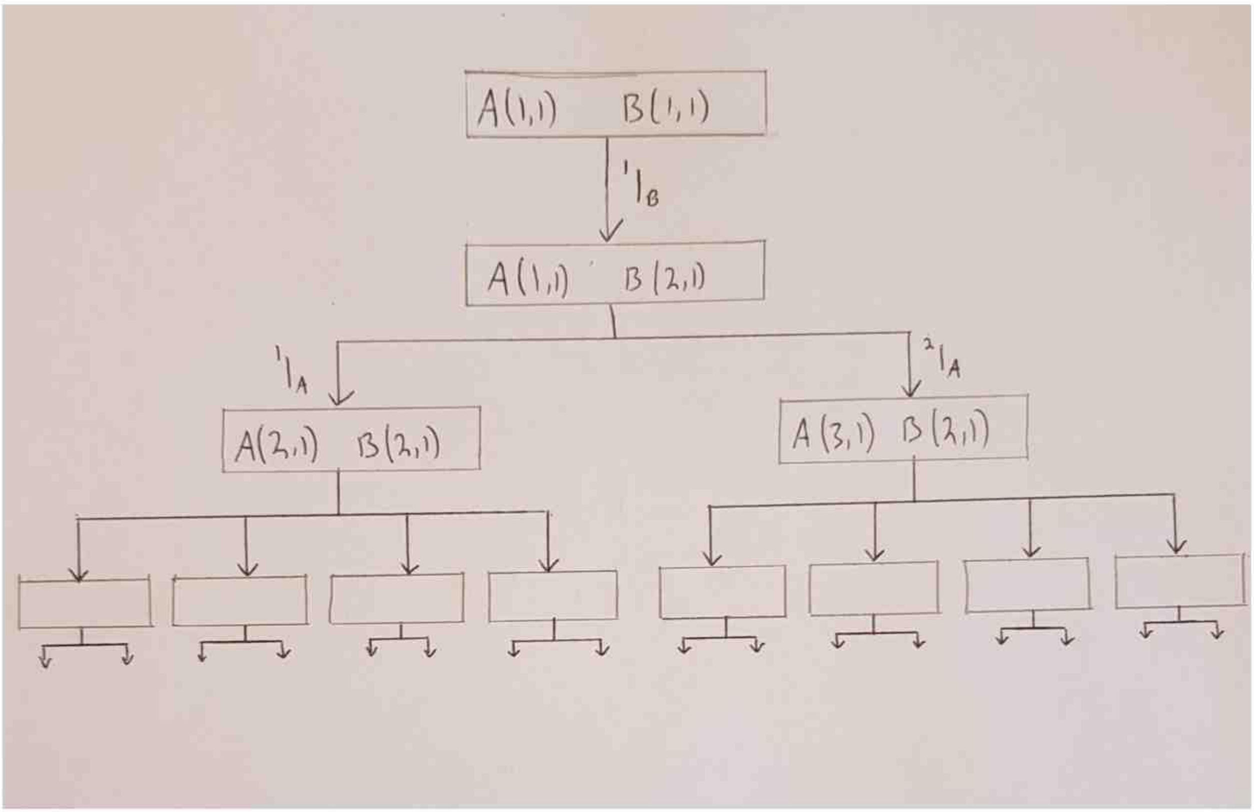
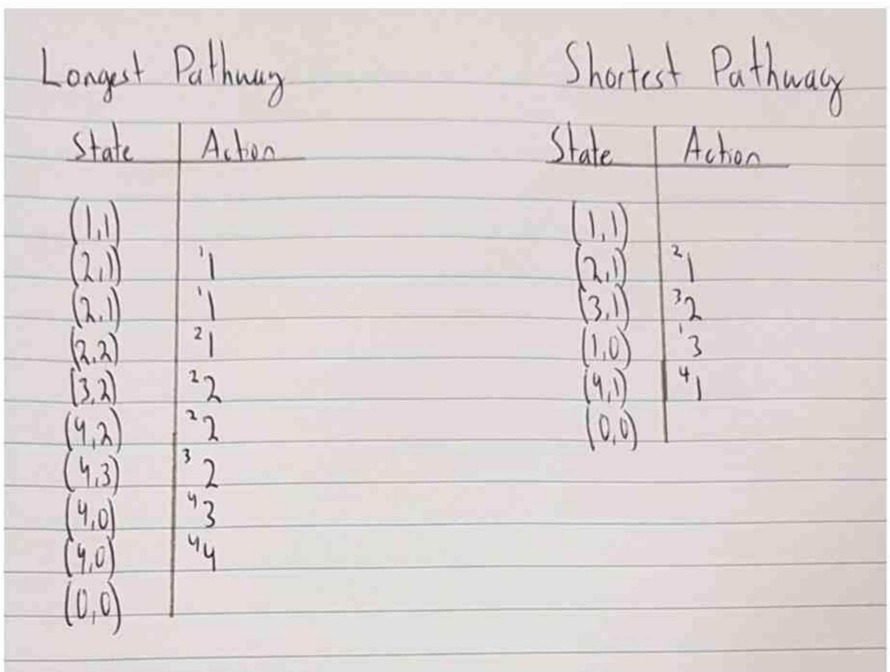


Figure 4: Game tree for the initial moves of the base game of Chopsticks

Every row would show the possible states depending on the actions from previous states. Considering that a hand can only be attacked and have its amount of fingers increased until its eventually outstretched and out of the game, the game should come to an end rather quickly and the game tree shouldn't expand much further than around a dozen rows.

We can already predict certain strategies to help understand the game better, such as the fastest and the slowest ways to win. To do so, we can classify the states of the game into ‘generations’, where each generation is the next row in the game tree diagram. We can also identify the ways to play the game as ‘pathways’ along the game tree. Thus, the longest pathway would then have the most amount of generations while the shortest pathway would have the least amount of generations. Since the longest pathway means taking the largest number of moves to outstretch your opponent’s hands, we can reason that this pathway consists of actions that do minimal damage. This would be to attack the opponent’s smaller hand using one’s own smaller hand. Using the same logic, the shortest pathway consists of maximizing the damage done.



Longest Pathway

State	Action
(1,1)	
(2,1)	$^{1}1$
(2,1)	$^{1}1$
(2,2)	$^{2}1$
(3,2)	$^{2}2$
(4,2)	$^{2}2$
(4,3)	$^{3}2$
(4,0)	$^{4}3$
(4,0)	$^{4}4$
(0,0)	

Shortest Pathway

State	Action
(1,1)	
(2,1)	$^{2}1$
(3,1)	$^{3}2$
(1,0)	$^{1}3$
(4,1)	$^{4}1$
(0,0)	

Figure 5: Longest and Shortest pathways written out

VI. Full Analysis of Basic Chopsticks

The game will be written out using Microsoft Excel leading to a different notation from the written notation. The first adjustment removes the action notations as they can be deduced from the changes in the game's states and including them would complicate the format of the tree diagram. The second change in notation is that I didn't include "A" nor "B" in front of the states of the game, instead putting the attacking hand on the left and the receiving hand on the right. I chose this because it would make the addition of states to find the following states consistent, reducing the risk of making many mistakes. Furthermore, instead of writing out the game tree diagram from top to bottom, I decided to write the tree diagram from left to right due to the short and wide cell shape in excel that makes horizontal stacking of information easier than vertical stacking.

Figure 6 displays the first few generations of the game.

	A	B	C	D	E	F
1	Gen	1	2	3	4	
2	Att. Ply.	A	B	A	B	
3					(2,2) (2,1)	
4					(3,1) (2,1)	
5				(2,1) (2,1)	(3,2) (2,1)	
6					(4,1) (2,1)	
7		(1,1) (1,1)	(2,1) (1,1)			
8					(2,2) (3,1)	
9				(3,1) (2,1)	(3,1) (3,1)	
10					(4,2) (3,1)	
11					(1,0) (3,1)	
12						

Figure 6: Initial game tree of Chopsticks

The arrows were added after screenshotting the excel sheet to clarify the tree diagram.

	A	B	C	D	E	F
1	Gen	1	2	3	4	
2	Att. Ply.	A	B	A	B	
3		(1,1) (1,1)	(2,1) (1,1)	(2,1) (2,1)	(2,2) (2,1)	
4					(3,1) (2,1)	
5					(3,2) (2,1)	
6					(4,1) (2,1)	
7						
8				(3,1) (2,1)	(2,2) (3,1)	
9					(3,1) (3,1)	
10					(4,2) (3,1)	
11					(1,0) (3,1)	
12						

Figure 7: Reformatted game tree of Chopsticks

This reformatting in figure 7 done so that the tree diagram expands downwards and rightwards, rather than before where it could expand both upwards and downwards. This is logical since there are no boundaries in the sheet towards the bottom nor the right, only boundaries at the top and on the left.

However, it is difficult start from the left and working your way right, as for you don't know how much space a single pathway needs. As such, instead of working from left to right, I will be writing down the entire game for the first row until it reaches its end and afterwards work my way inwards from right to left. Figure 8 below shows the beginning of the game tree of Chopsticks.

	A	B	C	D	E	F	G	H	I	J
1	A	B	A	B	A	B	A	B	A	B
2										
3	All									
4										
5	(1,1) (1,1)	(2,1) (1,1)	(2,1) (2,1)	(2,2) (2,1)	(3,2) (2,2)	(4,2) (3,2)	(4,3) (4,2)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)
6									(3,0) (4,0)	(0,0) (3,0)
7										
8								(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)
9									(3,0) (2,0)	(0,0) (3,0)
10										
11							(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)	
12								(2,0) (3,0)	(0,0) (2,0)	
13										
14							(2,0) (4,2)	(4,4) (2,0)	(0,0) (4,0)	
15								(2,0) (2,0)	(4,0) (2,0)	(0,0) (4,0)
16										
17						(2,0) (3,2)	(4,3) (2,0)	(0,0) (4,3)		
18							(2,0) (2,0)	(4,0) (2,0)	(0,0) (4,0)	
19										
20					(4,1) (2,2)	(3,2) (4,1)	(4,3) (3,2)	(3,0) (4,3)	(4,0) (3,0)	(0,0) (3,0)
21									(3,0) (3,0)	(0,0) (3,0)
22										
23								(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)
24									(3,0) (2,0)	(0,0) (3,0)

Figure 8: Beginning of the game tree of Chopsticks

I started out fully expanding out line 5, until I reached a dead end at cell J5. Then, I took a step back to cell I5 to see if there are any other states I could get to besides [(0,0) (4,0)], which there wasn't. Thus, I took another step back to cell H5 and found another possibility, which was cell I6. This process of fully exploring pathways and taking steps back continued until the entire game was written out.

The longest pathway I found was 13 generations long, while the shortest pathway was 6 generations long. While the shorter pathway followed the method I predicted, the longer pathway did not. Figure 10 shows the longest pathway using the two-player notation I made.

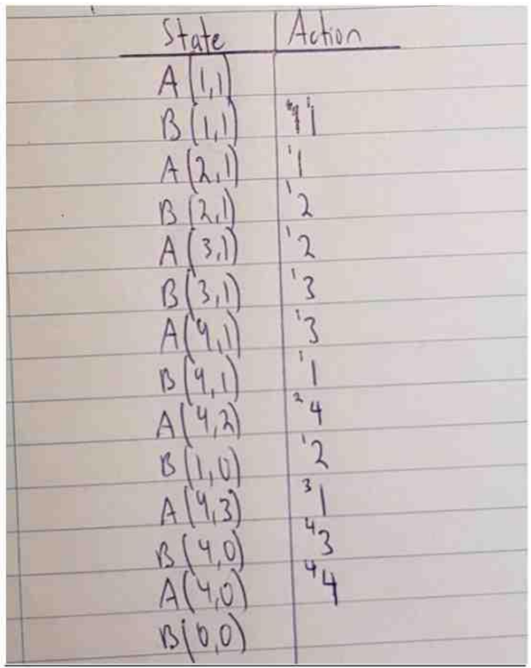


Figure 9: Written notation of longest pathway

The concept of attacking with the lowest value still holds, however rather than attacking the opponent's lowest value they instead consistently attack one hand. This allows each player to keep one hand with a single finger, thus sustaining weak attacks. This can be seen under the Action column as the superscripted number in figure 10 above, which indicates the attacking value, is consistently a 1. This continues until both hands are close to being outstretched, after which it goes on to try and preserve the lowest possible attacks on either side which soon ends the game. Therefore, the longest pathway functions by minimizing the damage done for as long as possible.

Once I had completed the tree diagram, the winning strategy still wasn't clear. Thus, I turned to color coding. The first step I took was to color all the sure-victories for player A and player B in green and red respectively, as seen in figure 11.

	A	B	C	D	E	F	G	H	I	J	K
1	A	B	A	B	A	B	A	B	A	B	
2											
3	All										
4											
5	(1,1) (1,1)	(2,1) (1,1)	(2,1) (2,1)	(2,2) (2,1)	(3,2) (2,2)	(4,2) (3,2)	(4,3) (4,2)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)	
6									(3,0) (4,0)	(0,0) (3,0)	
7											
8								(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)	
9									(3,0) (2,0)	(0,0) (3,0)	
10											
11							(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)		
12								(2,0) (3,0)	(0,0) (2,0)		
13											
14							(2,0) (4,2)	(4,4) (2,0)	(0,0) (4,0)		
15								(2,0) (2,0)	(4,0) (2,0)	(0,0) (4,0)	
16											
17						(2,0) (3,2)	(4,3) (2,0)	(0,0) (4,3)			
18							(2,0) (2,0)	(4,0) (2,0)	(0,0) (4,0)		
19											
20					(4,1) (2,2)	(3,2) (4,1)	(4,3) (3,2)	(3,0) (4,3)	(4,0) (3,0)	(0,0) (3,0)	
21									(3,0) (3,0)	(0,0) (3,0)	
22											
23								(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)	
24									(3,0) (2,0)	(0,0) (3,0)	
25											
26							(4,4) (3,2)	(3,0) (4,4)	(4,0) (3,0)	(0,0) (4,0)	
27								(2,0) (4,4)	(4,0) (2,0)	(0,0) (4,0)	
28											

Figure 10: Initially color coded game tree for Chopsticks

This meant that if the game is in one of the green states, then even if both player's consequential decisions are completely random, player A is certainly going to win. The orange colored states indicate states in which the outcome of the match still depends on the decision of the player.

To find the winning strategy from here, I assumed that both players play perfectly. This means that in any situation where they have to make a decision, they always make the decision which is the most favorable to them (Allis, Louis Victor. 1994.). This means that for those orange boxes, whoever's turn it is will choose a pathway that leads to their win. Thus, starting from right and working my way left, I recolored the orange boxes to dark green or dark red depending on who would make the decision as seen in figure 12.

Figure 11: Fully color coded game tree of Chopsticks

In the case of certain states, such as cells C5 and E5, the boxes were colored red even though it was player A's turn to decide. This is because as seen in cell E5, the two choices for player A both would result in player B's win, and thus the state of cell E5 would also result in player B's win and was thereby colored in red.

Looking at the same figure 12 above, the winning strategy also immediately becomes clear. From the start of the game, if both players play perfectly, then the first player that receives an attack will win the game. Moreover, even if player A plays randomly, as long as player B makes the right decisions at the dark red cells he will certainly win. There are, however, many different ways to win which all vary, since although player A loses he can still decide how he is going to lose. All of these pathways, however, end barely a couple of states after one of the opponent's hands are eliminated. This doesn't always apply; you can still lose after eliminating one of the opponent's hands. However, if both of your hands are capable of eliminating the remaining hand of your opponent, then you are certainly going to win.

Thus, the winning strategy of the game can be summarized as accumulating fingers and eliminating the opponent's hand as soon as possible, such that you are in the state of having two hands which can both eliminate the opponent's remaining hand.

VII. Expanding the rules of the game

The base game of Chopsticks is relatively simple and quick. There isn't much strategy to it, and it doesn't take long to master the game and play perfectly. Thus, I will be adjusting the game's rules that will expand upon the strategy of the game, after which I will find and analyze the new winning strategy.

The expanded version of Chopsticks will still use the same rules and goal of eliminating the opponent as before, though with two new game elements: remainders and transfers. Remainders means that the remaining fingers after eliminating a hand will revive that hand with that amount of fingers. A demonstration is show in figure 13.

Figure 12: Demonstration of remainder rule

If a hand with 4 fingers is attacked by a hand with 3 fingers, rather than being eliminated after reaching 5 fingers, the hand is in play again with a value equal to the remainder 2. Thus, to eliminate a hand and remove it from the game the hand's fingers must amount to exactly 5.

The other rule, transferring, is an alternative action to attacking. Rather than attacking during your turn, you have the option to change the configuration of fingers on your hands. Essentially, you are sending fingers from one hand to the other. An example is show in figure 14.

Figure 13: Demonstration of transfer rule

If my hand configuration is (4,1), I can choose to transfer to (3,2) and give up my attacking turn. Furthermore, transferring can also be done to bring back an eliminated hand. So if my hand configuration is (4,0), I can choose to transfer to either (3,1) or (2,2). However, in order to prevent continuous transfers, it is not allowed to mirror hand configurations (such as transferring from (3,1) to (1,3)), nor is it allowed to transfer back (such as transferring from (4,1) to (3,2), and in the following turn transferring back to (4,1)).

VIII. Analyzing the Expanded Game

Because of the new additions to the game which allow for more actions and restrict the ways you can eliminate the opponent, I'm expecting the generations of the game to be longer than they were in the base game. Plotting out the pathways will be done similarly to before, though due to the greater expected possibilities, I won't be repeating states I've explored which are highlighted in red. The green cells indicate victories.

B(3,3) A(2,0)	A(0,0) B(3,3)					
	A(2,0) B(4,2)	B(2,1) A(2,0)	A(4,0) B(2,1)			
			A(3,0) B(2,1)			
			A(2,0) B(3,0)	B(0,0) A(2,0)		
				B(3,0) A(1,1)		
		B(4,4) A(2,0)	A(1,0) B(4,4)	B(4,0) A(1,0)		
			A(2,0) B(3,0)			
			A(2,0) B(2,1)			
		B(4,2) A(1,1)	A(1,0) B(4,2)	B(2,0) A(1,0)		
				B(4,3) A(1,0)		
			A(3,1) B(4,2)	B(2,2) A(3,1)	A(1,0) B(2,2)	B(3,2) A(1

Figure 14: Extract from expanded Chopsticks game tree

The reasoning behind the discrepancy in game trees is due to the addition of transferring, since transferring makes repeating incredibly common. In the base game, one state would lead to a potential four other states, though this would only be one-way as for none of those four other states could lead back to the original state. In the expanded game, however, besides the four uni-directional options, there are a couple of other possible states due to transferring. The crucial thing is that these possible states due to transfers aren't uni-directional, but work both ways. This means that rather than having the game tree diagram be a directed graph, which is a graph with uni-directional edges like with the base game (Chartrand, Gary. 1978.), the pathways of the expanded game instead look like an interlinked network, of which a simplified version can be seen in.

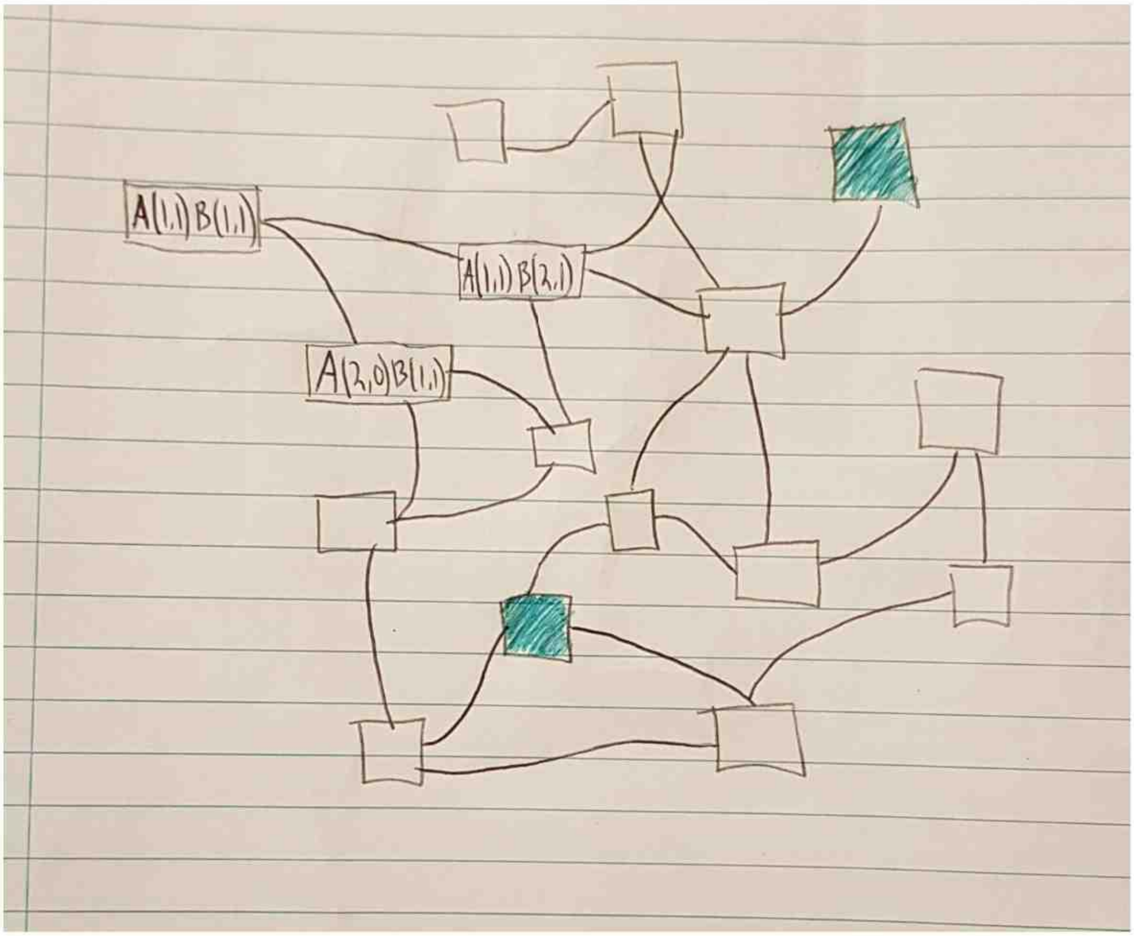


Figure 15: Simplified visual representation of the expanded Chopstick's 'game tree'

The nodes above represent the different states of the game, of which the green ones are wins, while the edges are the actions. Soon after the starting state in the top left, you enter the network which essentially covers every single possible state of the game. Through the right actions, you can reach any state from any state. The question is, how does this affect the winning strategy?

Essentially, there is no winning strategy. If both players play perfectly, then the game would just lead around in circles forever within the network without coming to an end. There are of course specific states which will lead to a certain win, which can be termed as 'traps'. These traps may only be a couple states long where the trap state quickly leads to the win state, though the more elaborate traps could take several states before leading to a win state. To a perfect player, these traps can simply be avoided by choosing a different action leading to a different state than it, since perfect players would be able to identify those traps and avoid them. Figure 17 below gives an overview of what the network looks like with traps shown outside.

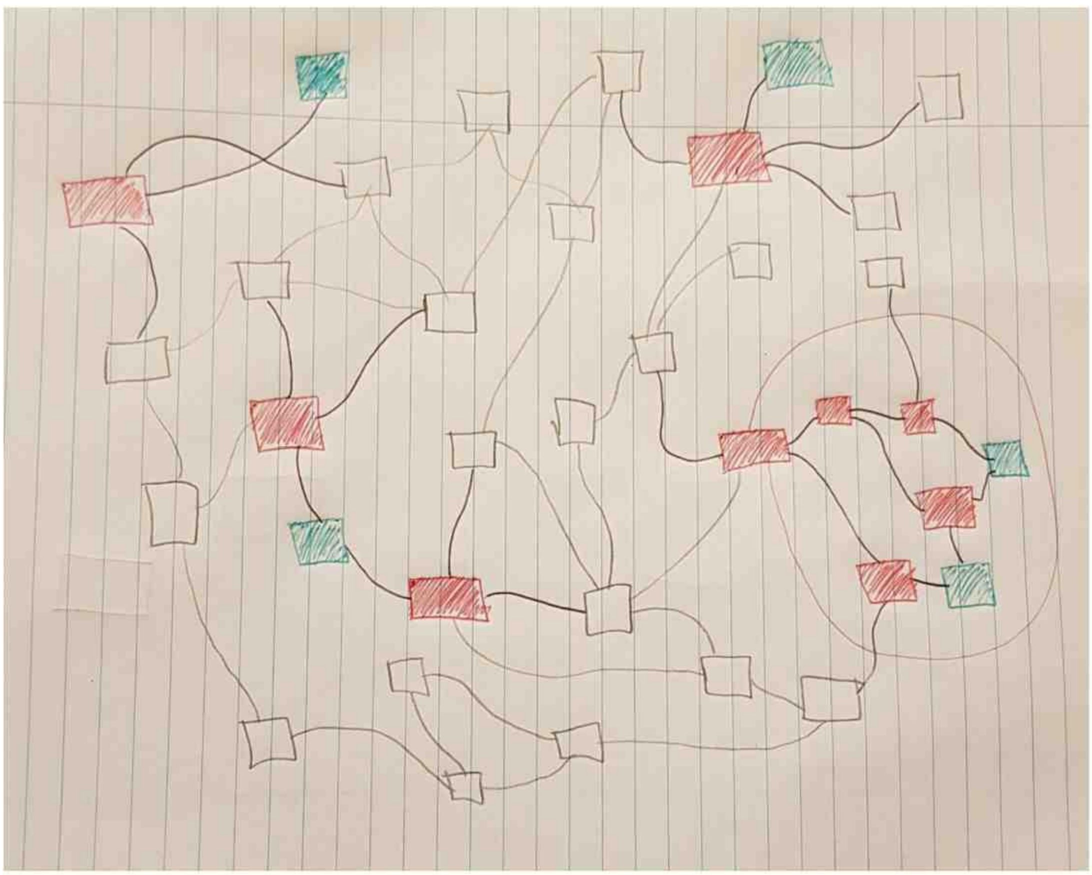


Figure 16: Simplified expanded chopstick's game tree, including traps

The traps from figure 17 highlighted in red, which lead to wins that are highlighted in green. The more elaborate trap is outlined in a red circle, where there are a few intermediate states before a win. Players have the chance to fall into these traps, but as seen in the diagram, they also have the possibility to escape back into the network before falling into the traps.

However, inexperienced players are more likely to fall into these traps. Furthermore, they are more likely to fall for the elaborate traps rather than the short ones since the short traps are more predictable and can be simulated within a few moves and thereby identified. The elaborate traps, however, aren't as predictable and thereby effective against these players. After looking through the expanded game diagram repeatedly, the most elaborate trap is seen in figure 18 below.

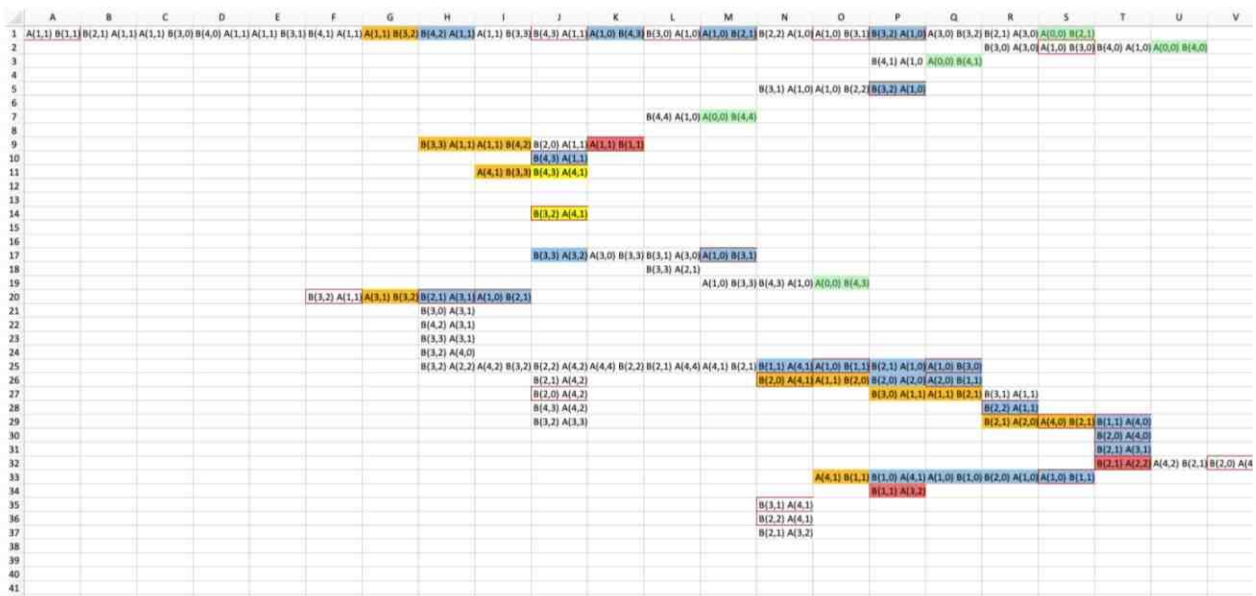


Figure 17: Game tree for expanded game showing a possible winning strategy using a trap

The orange boxes represent decisions which affect the outcome, blue boxes represent certain victory for player B if he makes the right choices, and the green boxes are player B's wins while the red boxes are player A's wins.

Therefore, we can see that there is no perfect winning strategy for the expanded game. At most, one can lead the opponent towards an elaborate trap and hope he falls into it. To do this, from the start of the game, player B setting up the trap must be the one getting hit first. Afterwards, the you transfer to have one hand with a 3. This forces player A to hit player B, as for performing a transfer to (2,0) would lead it to being eliminated due to player B's (3,0). This continues until cell G1, where player A makes the deciding move on whether to fall for the trap or not. Player A's option is to either hit player B to make a (4,2) or a (3,3). The B(4,2) A(1,1) state would lead to player B's victory, while the B(3,3) A(1,1) state would lead back into the complicated network. Figure 18 from before shows a few possible ways to try and get the win if player A makes the (3,3) choice, though the chances of this leading to a win are low.

IX. Conclusion

The two versions of Chopsticks I investigated were the base game and the expanded game. The base game was solved with a weak proof, meaning that there is an algorithm which can secure a win regardless of the opponent's moves from the beginning of the game onwards. This was done by generating the entire game tree for the base game, and extracting the winning strategy. The winning strategy for the base game involved accumulating fingers on your own hands and eliminating one of the opponent's hands as soon as possible.

The expanded version of the game added the elements of transfers and remainders. The expanded game was not solved, and only a possibility to win against non-perfect-play opponents was provided. The transferring element of the game altered the entire game tree, turning into an interlinked network rather than a branching out network. The winning strategy didn't exist for this game considering perfect play, though the method to set up the most elaborate trap was found which involved getting to the state of (1,1) (3,2). The network was too complex to solve it within this investigation, though it should be possible to solve the game once all pathways have not only been explored, but especially reorganized such that the data is accessible and readable.

Besides the expanded version using transfers and remainders, the game can further be expanded to include more fingers per hand, more hands, and more players. The addition of more hand and more fingers would expectedly expand the game trees and networks that are currently representing the game. More players, however, would redefine the game and new rules would have to be added and adjusted.

X. **Evaluation**:

The biggest obstacle I came across during the investigation was the unexpected complication due to the expansion of the game. This is because the complication prevented me from solving the game. Originally, the plan was to start by analyzing the expanded chopsticks and then take it further to adding more players. However, analyzing the expanded chopsticks was already a massive challenge in itself and analyzing an even more complex version would be overkill. Thus, I referred back to the base game, which did work out well and was nicely solved.

The winning strategy for the base game works flawlessly, as long as the player ensures he is the first one to be attacked and follows the winning pathways. However, should he make a mistake and choose the wrong action, the opponent may have the chance to win. The winning strategy for the expanded game, on the other hand, is lacking. Considering perfect play there is no winning strategy, which means that for the method I found to work the opponent must make an unfavorable decision. Furthermore, besides the winning strategy I didn't manage to find many other algorithms or strategies, such as how to avoid traps or how to set up other traps besides the one I showed.

XI. Works Cited:

Allis, Louis Victor. "Searching for Solutions in Games and Artificial Intelligence." *Rijksuniversiteit Limburg te Maastricht*, 1994.

Chartrand, Gary. *Introductory Graph Theory*. Dover Publications, Inc., 1978.

Chein, Michel. "10.3 Backwards Chaining." *Graph-Based Knowledge Representation: Computational Foundations of Conceptual Graphs*, edited by Marie-Laure Mugnier, Springer-Verlag London Limited, 2009, pp. 291–295.

"Chopsticks (Hand Game)." *Revolvy*, Revolvy, 17 Aug. 2018, www.revolvy.com/page/Chopsticks-%28hand-game%29.

[redacted]

"Game Theory I: Extensive Form." *Policonomics*, Policonomics, 2018, policonomics.com/lp-game-theory1-extensive-form/.

"Japanese Games – Chopsticks (Hand Game)." *Sakura's Zasshi*, Wordpress, 16 Oct. 2008, sakuraentorizasshi.wordpress.com/2008/10/16/japanese-games-chopsticks-hand-game/.

XII. **Appendix**

Figure 3 below was an attempt at trying to write the notation in terms of expressions and equations.

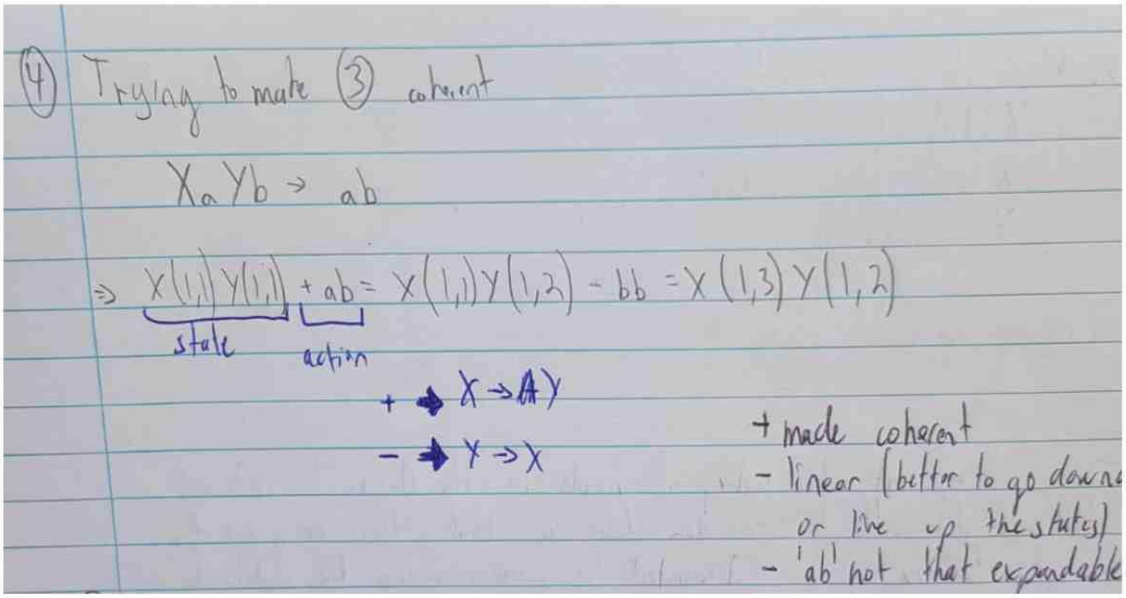


Figure 18

Figure 4 below shows an attempt at trying the notation using matrices.

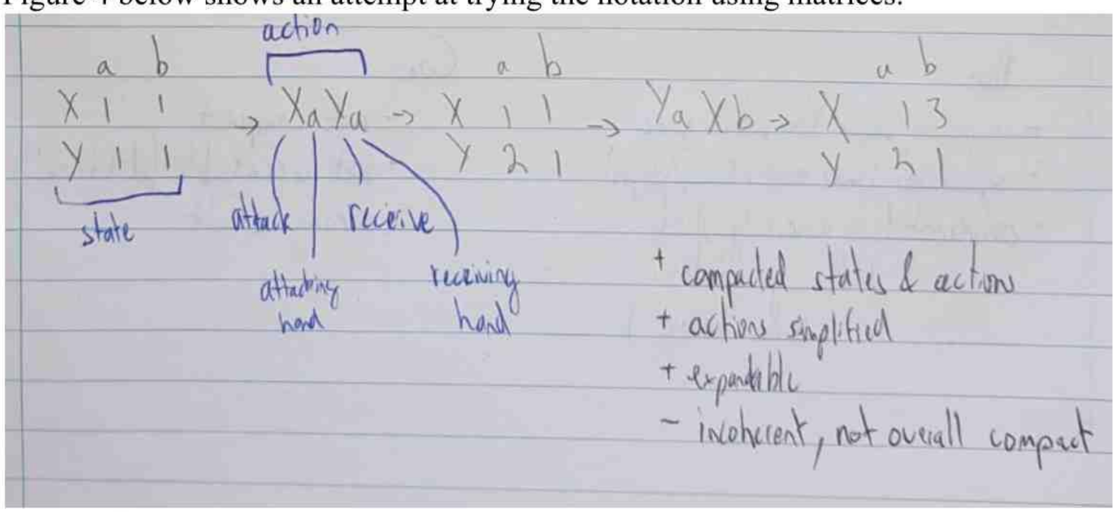


Figure 19

Neither of these notations worked out in the end

In figure 16 below and figure 17 on the following page, zoomed out overviews of the expanded game compared to the base game respectively are shown. The expanded game diagram (figure 16) makes its way diagonal and only goes back a few states every now and then before erratically shooting forwards again, as compared to the base game (figure 17) which is structured and coherent. To put things in perspective, the red box on figure 16 is the part that was extracted and displayed in figure 15 above. The red box on figure 17 is the part that was extracted and displayed ins figure 8 from part V.

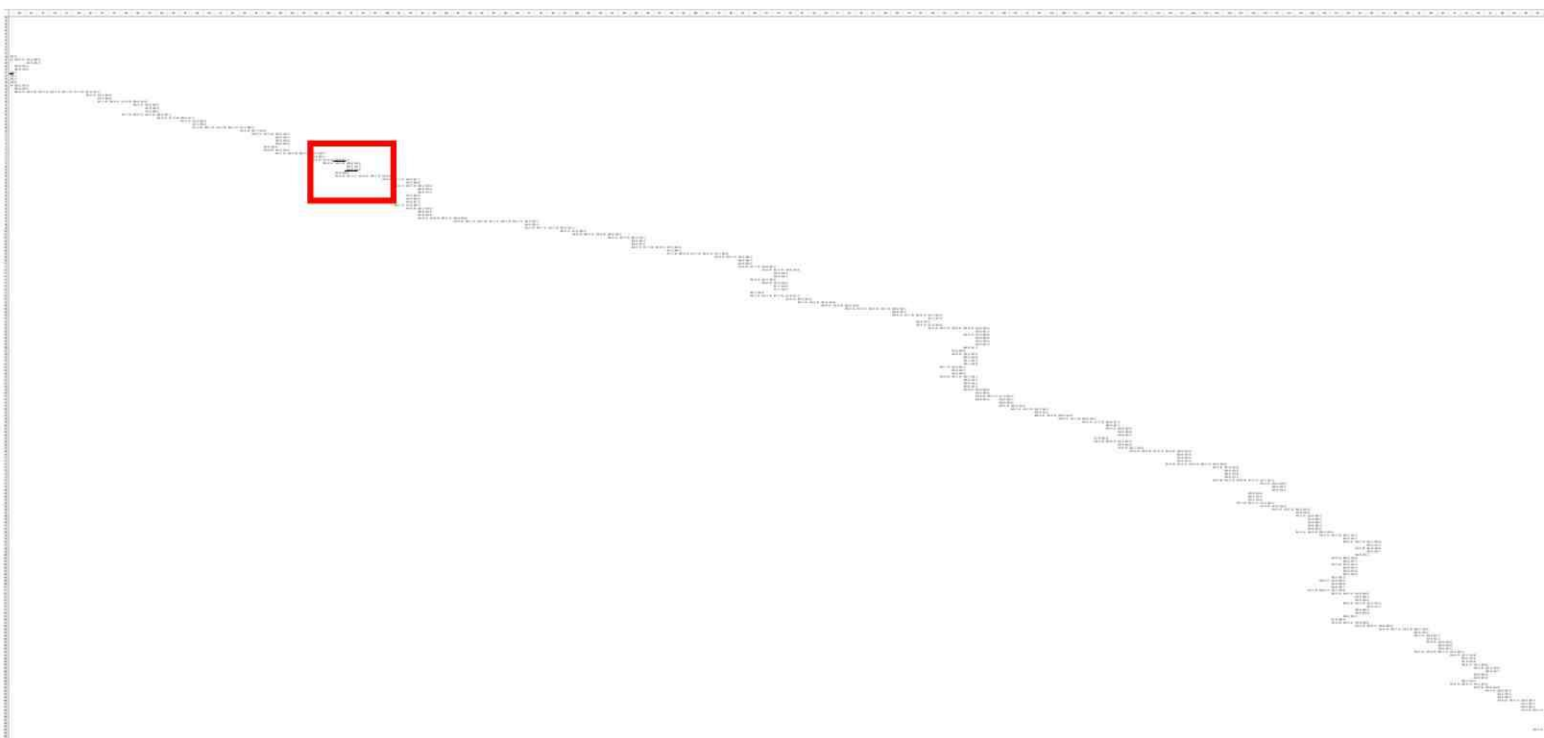


Figure 20: Zoomed out overview of the expanded Chopsticks game tree

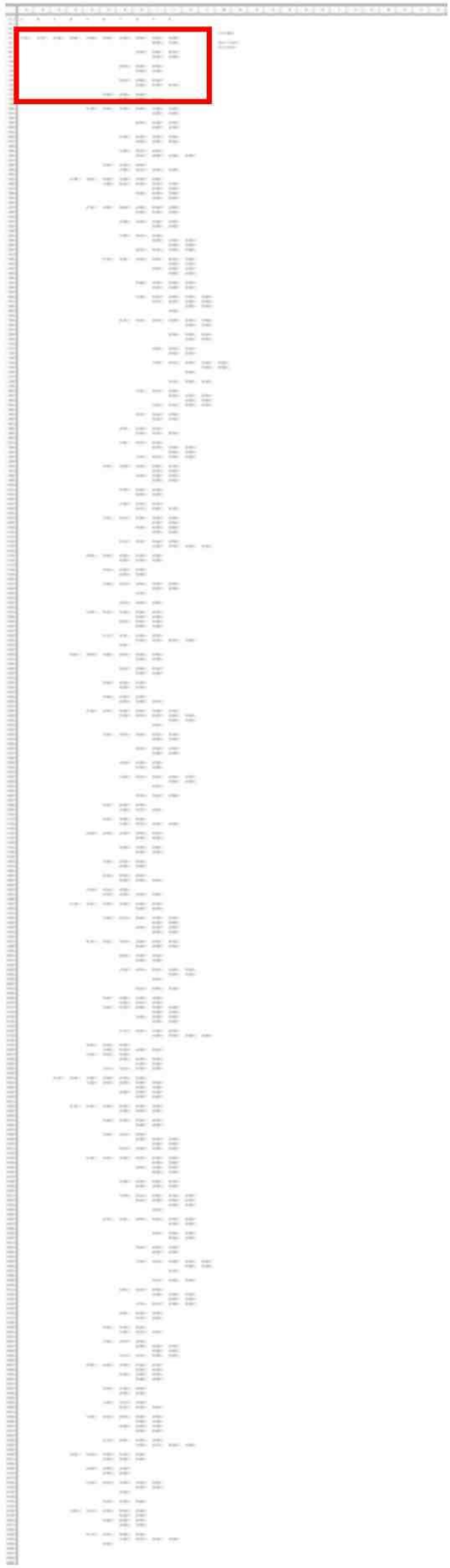


Figure 21: Basic Chopsticks game tree, zoomed out for full overview

<u>Game tree of the base game</u>

	A	B	C	D	E	F	G	H	I	J	K	L
1	A	B	A	B	A	B	A	B	A	B		
2												
3	All											
4												
5	(1,1) (1,1)	(2,1) (1,1)	(2,1) (2,1)	(2,2) (2,1)	(3,2) (2,2)	(4,2) (3,2)	(4,3) (4,2)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)		
6									(3,0) (4,0)	(0,0) (3,0)		
7												
8								(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)		
9									(3,0) (2,0)	(0,0) (3,0)		
10												
11							(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)			
12								(2,0) (3,0)	(0,0) (2,0)			
13												
14							(2,0) (4,2)	(4,4) (2,0)	(0,0) (4,0)			
15								(2,0) (2,0)	(4,0) (2,0)	(0,0) (4,0)		
16												
17						(2,0) (3,2)	(4,3) (2,0)	(0,0) (4,3)				
18							(2,0) (2,0)	(4,0) (2,0)	(0,0) (4,0)			
19												
20					(4,1) (2,2)	(3,2) (4,1)	(4,3) (3,2)	(3,0) (4,3)	(4,0) (3,0)	(0,0) (3,0)		
21									(3,0) (3,0)	(0,0) (3,0)		
22												
23								(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)		
24									(3,0) (2,0)	(0,0) (3,0)		
25												
26							(4,4) (3,2)	(3,0) (4,4)	(4,0) (3,0)	(0,0) (4,0)		
27								(2,0) (4,4)	(4,0) (2,0)	(0,0) (4,0)		
28												
29							(1,0) (3,2)	(4,2) (1,0)	(0,0) (4,2)			
30								(3,3) (1,0)	(4,0) (3,3)	(3,0) (4,0)	(0,0) (3,0)	
31												
32						(2,0) (4,1)	(4,3) (2,0)	(0,0) (4,3)				
33							(1,0) (2,0)	(3,0) (1,0)	(4,0) (3,0)	(0,0) (4,0)		
34												
35				(3,1) (2,1)	(2,2) (3,1)	(3,3) (2,2)	(2,0) (3,3)	(3,0) (2,0)	(0,0) (3,0)			
36						(1,0) (2,2)	(3,2) (1,0)	(3,0) (3,2)	(3,0) (3,0)	(0,0) (3,0)		
37									(2,0) (3,0)	(0,0) (2,0)		
38								(4,0) (3,2)	(3,0) (4,0)	(0,0) (3,0)		
39									(2,0) (4,0)	(0,0) (2,0)		
40												
41					(3,1) (3,1)	(3,2) (3,1)	(3,3) (3,2)	(3,0) (3,3)	(3,0) (3,0)	(0,0) (3,0)		
42								(2,0) (3,3)	(3,0) (2,0)	(2,0) (3,0)		
43												
44							(4,3) (3,2)	(3,0) (4,3)	(4,0) (3,0)	(0,0) (3,0)		
45									(3,0) (3,0)	(0,0) (3,0)		
46												

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
44							(4,3) (3,2)	(3,0) (4,3)	(4,0) (3,0)	(0,0) (3,0)				
45									(3,0) (3,0)	(0,0) (3,0)				
46														
47							(1,0) (3,2)	(4,2) (1,0)	(0,0) (4,2)					
48									(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)			
49										(2,0) (3,0)	(0,0) (2,0)			
50								(3,3) (1,0)	(4,0) (3,3)	(3,0) (4,0)	(0,0) (3,0)			
51														
52						(4,1) (3,1)	(3,2) (4,1)	(4,3) (3,2)	(3,0) (4,3)	(4,0) (3,0)	(0,0) (4,0)			
53										(3,0) (3,0)	(0,0) (3,0)			
54									(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)			
55										(3,0) (2,0)	(0,0) (4,0)			
56														
57								(4,4) (3,2)	(3,0) (4,4)	(4,0) (3,0)	(0,0) (4,0)			
58									(2,0) (4,4)	(4,0) (2,0)	(0,0) (4,0)			
59														
60								(1,0) (3,2)	(3,3) (1,0)	(4,0) (3,3)	(3,0) (4,0)	(0,0) (3,0)		
61									(4,2) (1,0)	(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)		
62											(2,0) (3,0)	(0,0) (2,0)		
63										(0,0) (4,2)				
64														
65							(4,1) (4,1)	(4,2) (4,1)	(4,3) (4,2)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)		
66											(3,0) (4,0)	(0,0) (3,0)		
67														
68										(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)		
69											(3,0) (2,0)	(0,0) (3,0)		
70														
71									(4,0) (4,2)	(4,0) (4,0)	(0,0) (4,0)			
72										(2,0) (4,0)	(0,0) (2,0)			
73														
74									(1,0) (4,2)	(4,3) (1,0)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)	
75												(3,0) (4,0)	(0,0) (3,0)	
76											(0,0) (4,3)			
77														
78										(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)		
79														
80								(1,0) (4,1)	(4,2) (1,0)	(0,0) (4,2)				
81										(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)		
82											(2,0) (3,0)	(0,0) (2,0)		
83									(1,0) (1,0)	(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)		
84														
85								(4,0) (4,1)	(4,0) (4,0)	(0,0) (4,0)				
86									(4,0) (1,0)	(0,0) (4,0)				
87														
88							(3,0) (4,1)	(4,4) (3,0)	(0,0) (4,4)					
89								(1,0) (3,0)	(4,0) (1,0)	(0,0) (4,0)				
90														
91							(1,0) (4,1)	(4,2) (1,0)	(0,0) (4,2)					

	A	B	C	D	E	F	G	H	I	J	K	L	M
89								(1,0) (3,0)	(4,0) (1,0)	(0,0) (4,0)			
90													
91							(1,0) (4,1)	(4,2) (1,0)	(0,0) (4,2)				
92									(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)		
93										(2,0) (3,0)	(0,0) (2,0)		
94								(1,0) (1,0)	(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)		
95													
96						(4,3) (3,1)	(4,3) (4,3)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)			
97									(3,0) (4,0)	(0,0) (3,0)			
98								(3,0) (4,3)	(4,0) (3,0)	(0,0) (4,0)			
99									(3,0) (3,0)	(0,0) (3,0)			
100													
101							(3,0) (4,3)	(4,0) (3,0)	(0,0) (4,0)				
102								(3,0) (3,0)	(0,0) (3,0)				
103													
104							(1,0) (4,3)	(4,4) (1,0)	(0,0) (4,4)				
105								(3,0) (1,0)	(4,0) (3,0)	(0,0) (4,0)			
106													
107						(1,0) (3,1)	(3,2) (1,0)	(3,0) (3,2)	(3,0) (3,0)	(0,0) (3,0)			
108									(2,0) (3,0)	(0,0) (2,0)			
109								(4,0) (3,2)	(3,0) (4,0)	(0,0) (3,0)			
110									(2,0) (4,0)	(0,0) (2,0)			
111													
112							(4,1) (1,0)	(2,0) (4,1)	(4,3) (2,0)	(0,0) (4,3)			
113									(1,0) (2,0)	(3,0) (1,0)	(4,0) (3,0)	(0,0) (4,0)	
114													
115					(4,2) (3,1)	(3,3) (4,2)	(4,0) (3,3)	(3,0) (4,0)	(0,0) (3,0)				
116							(2,0) (3,3)	(3,0) (2,0)	(0,0) (3,0)				
117													
118						(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)					
119							(2,0) (3,0)	(0,0) (2,0)					
120													
121						(1,0) (4,2)	(4,3) (1,0)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)			
122									(3,0) (4,0)	(0,0) (3,0)			
123								(0,0) (4,3)					
124													
125							(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)				
126													
127					(1,0) (3,1)	(3,2) (1,0)	(3,0) (3,2)	(3,0) (3,0)	(0,0) (3,0)				
128								(2,0) (3,0)	(0,0) (2,0)				
129							(4,0) (3,2)	(3,0) (4,0)	(0,0) (3,0)				
130								(2,0) (4,0)	(0,0) (2,0)				
131													
132						(4,1) (1,0)	(2,0) (4,1)	(4,3) (2,0)	(0,0) (4,3)				
133								(1,0) (2,0)	(3,0) (1,0)	(4,0) (3,0)	(0,0) (4,0)		
134							(0,0) (4,1)						
135													
136				(3,2) (2,1)	(3,2) (3,2)	(4,3) (3,2)	(3,0) (4,3)	(4,0) (3,0)	(0,0) (3,0)				

	A	B	C	D	E	F	G	H	I	J	K	L
133								(1,0) (2,0)	(3,0) (1,0)	(4,0) (3,0)	(0,0) (4,0)	
134							(0,0) (4,1)					
135												
136				(3,2) (2,1)	(3,2) (3,2)	(4,3) (3,2)	(3,0) (4,3)	(4,0) (3,0)	(0,0) (3,0)			
137								(3,0) (3,0)	(0,0) (3,0)			
138												
139							(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)			
140								(3,0) (2,0)	(0,0) (3,0)			
141												
142						(3,0) (3,2)	(3,0) (3,0)	(0,0) (3,0)				
143							(2,0) (3,0)	(0,0) (2,0)				
144												
145						(2,0) (3,2)	(4,3) (2,0)	(0,0) (4,3)				
146							(2,0) (2,0)	(4,0) (2,0)	(0,0) (4,0)			
147												
148					(4,1) (3,2)	(3,3) (4,1)	(4,4) (3,3)	(3,0) (4,4)	(4,0) (3,0)	(0,0) (4,0)		
149							(1,0) (3,3)	(4,3) (1,0)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)	
150										(3,0) (4,0)	(0,0) (3,0)	
151									(0,0) (4,3)			
152												
153						(4,2) (4,1)	(4,3) (4,2)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)		
154									(3,0) (4,0)	(0,0) (3,0)		
155												
156								(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)		
157									(3,0) (2,0)	(0,0) (3,0)		
158												
159							(4,0) (4,2)	(4,0) (4,0)	(0,0) (4,0)			
160								(2,0) (4,0)	(0,0) (2,0)			
161												
162							(1,0) (4,2)	(4,3) (1,0)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)	
163										(3,0) (4,0)	(0,0) (3,0)	
164									(0,0) (4,3)			
165												
166								(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)		
167												
168						(3,0) (4,1)	(4,4) (3,0)	(0,0) (4,4)				
169							(1,0) (3,0)	(4,0) (1,0)	(0,0) (4,0)			
170												
171						(2,0) (4,1)	(4,3) (2,0)	(0,0) (4,3)				
172							(1,0) (2,0)	(3,0) (1,0)	(4,0) (3,0)	(0,0) (4,0)		
173												
174					(4,2) (3,2)	(4,3) (4,2)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)			
175								(3,0) (4,0)	(0,0) (3,0)			
176												
177							(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)			
178								(3,0) (2,0)	(0,0) (3,0)			
179												
180						(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)				

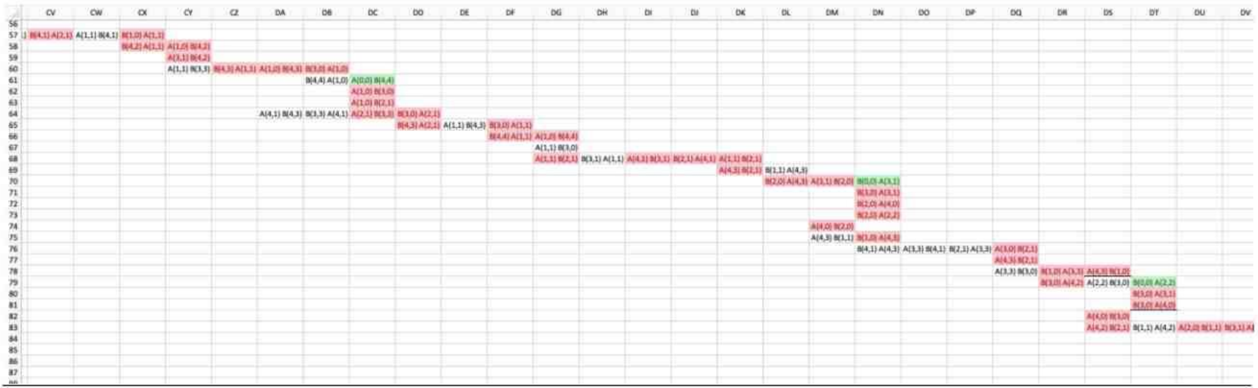
	A	B	C	D	E	F	G	H	I	J	K	L	M
178								(3,0) (2,0)	(0,0) (3,0)				
179													
180						(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)					
181							(2,0) (3,0)	(0,0) (2,0)					
182													
183						(2,0) (4,2)	(4,4) (2,0)	(0,0) (4,0)					
184							(2,0) (2,0)	(4,0) (2,0)	(0,0) (4,0)				
185													
186					(1,0) (3,2)	(4,2) (1,0)	(0,0) (4,2)						
187						(3,3) (1,0)	(4,0) (3,3)	(3,0) (4,0)	(0,0) (3,0)				
188													
189				(4,1) (2,1)	(2,2) (4,1)	(4,3) (2,2)	(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)				
190								(3,0) (2,0)	(0,0) (3,0)				
191													
192						(1,0) (2,2)	(3,2) (1,0)	(3,0) (3,2)	(3,0) (3,0)	(0,0) (3,0)			
193									(2,0) (3,0)	(0,0) (2,0)			
194								(4,0) (3,2)	(3,0) (4,0)	(0,0) (3,0)			
195									(2,0) (4,0)	(0,0) (2,0)			
196													
197					(3,1) (4,1)	(4,2) (3,1)	(3,3) (4,2)	(4,0) (3,3)	(3,0) (4,0)	(0,0) (3,0)			
198								(2,0) (3,3)	(3,0) (2,0)	(0,0) (3,0)			
199													
200							(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)				
201								(2,0) (3,0)	(0,0) (2,0)				
202													
203							(1,0) (4,2)	(4,3) (1,0)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)		
204										(3,0) (4,0)	(0,0) (3,0)		
205									(0,0) (4,3)				
206													
207								(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)			
208													
209						(4,4) (3,1)	(3,0) (4,4)	(4,0) (3,0)	(0,0) (4,0)				
210							(1,0) (4,4)	(4,0) (1,0)	(0,0) (4,0)				
211						(1,0) (3,1)	(3,2) (1,0)	(3,0) (3,2)	(3,0) (3,0)	(0,0) (3,0)			
212									(2,0) (3,0)	(0,0) (2,0)			
213								(4,0) (3,2)	(3,0) (4,0)	(0,0) (3,0)			
214									(2,0) (4,0)	(0,0) (2,0)			
215													
216							(4,1) (1,0)	(2,0) (4,1)	(4,3) (2,0)	(0,0) (4,3)			
217									(1,0) (2,0)	(3,0) (1,0)	(4,0) (3,0)	(0,0) (4,0)	
218													
219					(2,0) (4,1)	(4,3) (2,0)	(0,0) (4,3)						
220						(1,0) (2,0)	(3,0) (1,0)	(4,0) (3,0)	(0,0) (4,0)				
221					(1,0) (4,1)	(4,2) (1,0)	(0,0) (4,2)						
222							(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)				
223								(2,0) (3,0)	(0,0) (2,0)				
224						(1,0) (1,0)	(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)				

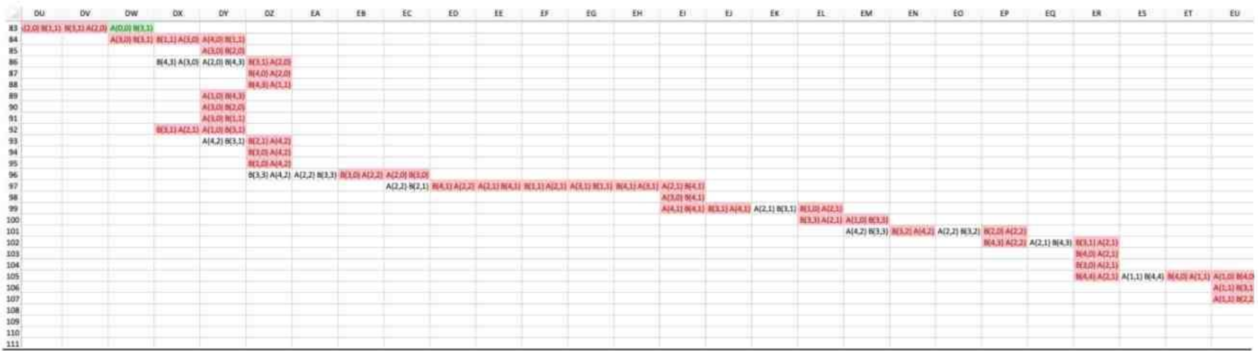
	A	B	C	D	E	F	G	H	I	J	K	L	M
223								(2,0) (3,0)	(0,0) (2,0)				
224						(1,0) (1,0)	(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)				
225													
226			(3,1) (2,1)	(2,2) (3,1)	(3,3) (2,2)	(2,0) (3,3)	(3,0) (2,0)	(0,0) (3,0)					
227					(1,0) (2,2)	(3,2) (1,0)	(3,0) (3,2)	(3,0) (3,0)	(0,0) (3,0)				
228								(2,0) (3,0)	(0,0) (2,0)				
229							(4,0) (3,2)	(3,0) (4,0)	(0,0) (3,0)				
230								(2,0) (4,0)	(0,0) (2,0)				
231													
232				(3,1) (3,1)	(3,2) (3,1)	(3,3) (3,2)	(3,0) (3,3)	(3,0) (3,0)	(0,0) (3,0)				
233							(2,0) (3,3)	(3,0) (2,0)	(2,0) (3,0)				
234													
235						(4,3) (3,2)	(3,0) (4,3)	(4,0) (3,0)	(0,0) (3,0)				
236								(3,0) (3,0)	(0,0) (3,0)				
237													
238						(1,0) (3,2)	(4,2) (1,0)	(0,0) (4,2)					
239								(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)			
240									(2,0) (3,0)	(0,0) (2,0)			
241							(3,3) (1,0)	(4,0) (3,3)	(3,0) (4,0)	(0,0) (3,0)			
242													
243					(4,1) (3,1)	(3,2) (4,1)	(4,3) (3,2)	(3,0) (4,3)	(4,0) (3,0)	(0,0) (4,0)			
244									(3,0) (3,0)	(0,0) (3,0)			
245								(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)			
246									(3,0) (2,0)	(0,0) (4,0)			
247													
248							(4,4) (3,2)	(3,0) (4,4)	(4,0) (3,0)	(0,0) (4,0)			
249								(2,0) (4,4)	(4,0) (2,0)	(0,0) (4,0)			
250													
251							(1,0) (3,2)	(3,3) (1,0)	(4,0) (3,3)	(3,0) (4,0)	(0,0) (3,0)		
252								(4,2) (1,0)	(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)		
253										(2,0) (3,0)	(0,0) (2,0)		
254									(0,0) (4,2)				
255													
256						(4,1) (4,1)	(4,2) (4,1)	(4,3) (4,2)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)		
257										(3,0) (4,0)	(0,0) (3,0)		
258													
259									(2,0) (4,3)	(4,0) (2,0)	(0,0) (4,0)		
260										(3,0) (2,0)	(0,0) (3,0)		
261													
262								(4,0) (4,2)	(4,0) (4,0)	(0,0) (4,0)			
263									(2,0) (4,0)	(0,0) (2,0)			
264													
265								(1,0) (4,2)	(4,3) (1,0)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)	
266											(3,0) (4,0)	(0,0) (3,0)	
267										(0,0) (4,3)			
268													
269									(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)		
270													

	A	B	C	D	E	F	G	H	I	J	K	L
268												
269									(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)	
270												
271							(1,0) (4,1)	(4,2) (1,0)	(0,0) (4,2)			
272									(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)	
273										(2,0) (3,0)	(0,0) (2,0)	
274								(1,0) (1,0)	(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)	
275												
276							(4,0) (4,1)	(4,0) (4,0)	(0,0) (4,0)			
277								(4,0) (1,0)	(0,0) (4,0)			
278												
279						(3,0) (4,1)	(4,4) (3,0)	(0,0) (4,4)				
280							(1,0) (3,0)	(4,0) (1,0)	(0,0) (4,0)			
281												
282						(1,0) (4,1)	(4,2) (1,0)	(0,0) (4,2)				
283								(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)		
284									(2,0) (3,0)	(0,0) (2,0)		
285							(1,0) (1,0)	(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)		
286												
287					(4,3) (3,1)	(4,3) (4,3)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)			
288								(3,0) (4,0)	(0,0) (3,0)			
289							(3,0) (4,3)	(4,0) (3,0)	(0,0) (4,0)			
290								(3,0) (3,0)	(0,0) (3,0)			
291												
292						(3,0) (4,3)	(4,0) (3,0)	(0,0) (4,0)				
293							(3,0) (3,0)	(0,0) (3,0)				
294												
295						(1,0) (4,3)	(4,4) (1,0)	(0,0) (4,4)				
296							(3,0) (1,0)	(4,0) (3,0)	(0,0) (4,0)			
297												
298					(1,0) (3,1)	(3,2) (1,0)	(3,0) (3,2)	(3,0) (3,0)	(0,0) (3,0)			
299								(2,0) (3,0)	(0,0) (2,0)			
300							(4,0) (3,2)	(3,0) (4,0)	(0,0) (3,0)			
301								(2,0) (4,0)	(0,0) (2,0)			
302												
303						(4,1) (1,0)	(2,0) (4,1)	(4,3) (2,0)	(0,0) (4,3)			
304								(1,0) (2,0)	(3,0) (1,0)	(4,0) (3,0)	(0,0) (4,0)	
305												
306				(4,2) (3,1)	(3,3) (4,2)	(4,0) (3,3)	(3,0) (4,0)	(0,0) (3,0)				
307						(2,0) (3,3)	(3,0) (2,0)	(0,0) (3,0)				
308												
309					(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)					
310						(2,0) (3,0)	(0,0) (2,0)					
311												
312					(1,0) (4,2)	(4,3) (1,0)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)			
313								(3,0) (4,0)	(0,0) (3,0)			
314							(0,0) (4,3)					
315												

	A	B	C	D	E	F	G	H	I	J	K	L
300							(4,0) (3,2)	(3,0) (4,0)	(0,0) (3,0)			
301								(2,0) (4,0)	(0,0) (2,0)			
302												
303						(4,1) (1,0)	(2,0) (4,1)	(4,3) (2,0)	(0,0) (4,3)			
304								(1,0) (2,0)	(3,0) (1,0)	(4,0) (3,0)	(0,0) (4,0)	
305												
306				(4,2) (3,1)	(3,3) (4,2)	(4,0) (3,3)	(3,0) (4,0)	(0,0) (3,0)				
307						(2,0) (3,3)	(3,0) (2,0)	(0,0) (3,0)				
308												
309					(3,0) (4,2)	(4,0) (3,0)	(0,0) (4,0)					
310						(2,0) (3,0)	(0,0) (2,0)					
311												
312					(1,0) (4,2)	(4,3) (1,0)	(4,0) (4,3)	(4,0) (4,0)	(0,0) (4,0)			
313								(3,0) (4,0)	(0,0) (3,0)			
314							(0,0) (4,3)					
315												
316						(2,0) (1,0)	(3,0) (2,0)	(0,0) (3,0)				
317												
318				(1,0) (3,1)	(3,2) (1,0)	(3,0) (3,2)	(3,0) (3,0)	(0,0) (3,0)				
319							(2,0) (3,0)	(0,0) (2,0)				
320						(4,0) (3,2)	(3,0) (4,0)	(0,0) (3,0)				
321							(2,0) (4,0)	(0,0) (2,0)				
322												
323					(4,1) (1,0)	(2,0) (4,1)	(4,3) (2,0)	(0,0) (4,3)				
324							(1,0) (2,0)	(3,0) (1,0)	(4,0) (3,0)	(0,0) (4,0)		
325						(0,0) (4,1)						
326												
327												
328												
329												
330												
331												

Game tree of the expanded game





	ET	EU	EV	EW	EX	EY	EZ	FA	FB	FC	FD	FE	FF	FG	FH	FI	FJ	FK	FL	FM	FN	FO	FP	FQ	FR	FS	FT
105	,0) A(1,1)	A(1,0) B(4,0)																									
106		A(1,1) B(3,1)																									
107		A(1,1) B(2,2)	B(3,2) A(1,1)	A(4,1) B(3,2)	B(2,2) A(4,1)	A(1,1) B(2,2)																					
108						A(4,3) B(2,2)	B(2,1) A(4,3)	A(3,1) B(2,1)																			
109								A(4,0) B(2,1)																			
110								A(3,0) B(2,1)																			
111								A(4,4) B(2,1)	B(1,1) A(4,4)	A(4,0) B(1,1)																	
112										A(4,4) B(2,0)	B(1,0) A(4,4)	A(4,0) B(1,0)															
113											B(2,0) A(3,0)																
114											B(2,0) A(2,1)																
115									B(2,0) A(4,4)	A(4,1) B(2,0)																	
116										A(4,4) B(1,1)	B(1,0) A(4,4)																
117											B(1,1) A(3,0)																
118											B(1,1) A(2,1)																
119									B(2,1) A(3,0)																		
120									B(2,1) A(2,1)	A(4,1) B(2,1)	B(1,1) A(4,1)	A(1,0) B(1,1)															
121												A(4,2) B(1,1)	B(1,0) A(4,2)														
122													B(3,1) A(4,2)	A(2,2) B(3,1)	B(1,0) A(2,2)												
123															B(3,3) A(2,2)	A(2,0) B(3,3)	B(3,0) A(2,0)										
124																	B(3,3) A(1,1)	A(4,1) B(3,3)	B(3,2) A(4,1)	A(2,1) B(3,2)	B(2,0) A(2,1)						
125																					B(4,3) A(2,1)						
126																					B(4,2) A(2,1)	A(1,1) B(4,2)	B(2,0) A(1,1)	A(3,1) B(2,0)			
127																								A(1,1) B(1,1)			
128																							B(4,3) A(1,1)				
129																							B(4,2) A(2,0)	A(1,0) B(4,2)			
130																								A(4,0) B(4,2)	B(3,2) A(4,0)	A(2,0) B(3,2)	B(2,0) A(2,0)
131																											
132																											B(4,3) A(2,0)
133																											
134																											
135																											
136																											B(3,2) A(1,1)
137																										A(1,0) B(3,2)	
138																										A(4,0) B(4,1)	B(3,1) A(4,0)
139																											B(4,0) A(4,0)
140																											B(4,1) A(3,1)
141																											B(4,1) A(2,2)
142																									B(4,1) A(4,0)	A(3,0) B(4,1)	
143																										A(0,0) B(4,1)	
144																										A(4,0) B(3,2)	
145																									B(4,2) A(3,1)	A(2,1) B(4,2)	B(2,1) A(2,1)
146																											B(4,4) A(2,1)
147																											B(2,0) A(2,1)
148																											B(4,3) A(2,1)
149																											B(4,2) A(3,0)
150																											

	FT	FU	FV	FW	FX	FY	FZ	GA	GB	GC	GD	GE	GF	GG	GH	GI	GJ	GK	GL	GM	GN	GO	GP	GQ	GR	GS	GT
129																											
130	0) A(2,0)	A(4,0) B(2,0)																									
131		A(2,0) B(1,1)																									
132	3) A(2,0)	A(1,0) B(4,3)																									
133		A(0,0) B(4,3)																									
134		A(2,0) B(2,0)																									
135		A(2,0) B(1,1)																									
136	2) A(1,1)																										
137																											
138	1) A(4,0)																										
139	0) A(4,0)																										
140	1) A(3,1)																										
141	1) A(2,2)																										
142																											
143																											
144																											
145	1) A(2,1)																										
146	4) A(2,1)																										
147	0) A(2,1)																										
148	3) A(2,1)																										
149	2) A(3,0)	A(2,0) B(4,2)																									
150		A(0,0) B(4,2)																									
151		A(3,0) B(3,1)	B(3,1) A(3,0)	A(1,0) B(3,1)																							
152		A(3,0) B(1,0)		A(4,0) B(3,1)																							
153				A(3,0) B(4,0)																							
154				A(3,0) B(2,2)	B(2,0) A(3,0)																						
155					B(2,2) A(2,1)	A(4,1) B(2,2)	B(2,1) A(4,1)																				
156							B(3,2) A(4,1)																				
157							B(2,2) A(3,2)	A(2,0) B(2,2)	B(4,2) A(2,0)																		
158									B(2,2) A(1,1)	A(3,1) B(2,2)	B(2,0) A(3,1)																
159											B(3,2) A(3,1)	A(1,1) B(3,2)	B(4,2) A(1,1)														
160													B(3,3) A(1,1)														
161													B(3,2) A(2,0)	A(0,0) B(3,2)													
162														A(4,0) B(3,2)													
163														A(2,0) B(4,1)													
164												A(1,0) B(3,2)															
165												A(4,3) B(3,2)	B(2,2) A(4,3)	A(3,1) B(2,2)													
166														A(4,0) B(2,2)													
167														A(4,3) B(3,1)	B(2,1) A(4,3)												
168															B(3,0) A(4,3)	A(3,2) B(3,0)	B(1,0) A(3,2)	A(4,2) B(1,0)	B(0,0) A(4,2)								
169																			B(3,0) A(4,2)								
170																			B(1,0) A(3,3)								
171																			B(1,0) A(1,0)								
172																		A(3,3) B(1,0)	B(4,0) A(3,3)	A(3,2) B(4,0)	B(2,0) A(3,2)	A(2,0) B(2,0)					
173																						A(4,3) B(2,0)	B(3,0) A(4,3)				
174																							B(0,0) A(4,3)				
175																							B(2,0) A(2,0)				
176																							B(2,0) A(1,1)				
177																						A(3,2) B(1,1)	B(4,1) A(3,2)	A(2,2) B(4,1)	B(1,1) A(2,2)	A(3,1) B(1,1)	
178																										A(2,2) B(2,0)	B(4,0) A(2,2)
179																											B(2,0) A(3,1)
180																											B(2,0) A(4,0)

		GO	GP	GQ	GR	GS	GT	GU	GV	GW	GX	GY	GZ	HA	HB	HC
172	,2)	A(2,0) B(2,0)														
173		A(4,3) B(2,0)	B(1,0) A(4,3)													
174			B(0,0) A(4,3)													
175			B(2,0) A(2,0)													
176			B(2,0) A(1,1)													
177		A(3,2) B(1,1)	B(4,1) A(3,2)	A(2,2) B(4,1)	B(1,1) A(2,2)	A(3,1) B(1,1)										
178						A(2,2) B(2,0)	B(4,0) A(2,2)									
179							B(2,0) A(3,1)									
180							B(2,0) A(4,0)									
181					B(4,3) A(2,2)											
182					B(4,1) A(3,1)											
183					B(4,1) A(4,0)											
184				A(3,1) B(4,1)	B(2,1) A(3,1)	A(1,0) B(2,1)										
185						A(3,3) B(2,1)	B(1,0) A(3,3)									
186							B(4,2) A(3,3)	A(3,2) B(4,2)	B(2,2) A(3,2)							
187									B(4,0) A(3,2)							
188									B(2,1) A(3,2)	A(2,0) B(2,1)						
189										A(4,3) B(2,1)						
190										A(4,2) B(2,1)						
191										A(3,3) B(2,1)						
192										A(3,2) B(3,0)						
193									B(4,4) A(3,2)	A(2,2) B(4,4)	B(4,1) A(2,2)					
194											B(4,4) A(3,1)	A(2,1) B(4,4)	B(4,1) A(2,1)			
195													B(4,0) A(2,1)			
196													B(4,4) A(3,0)	A(2,0) B(4,4)	B(4,1) A(2,0)	
197															B(4,4) A(1,1)	
198														A(3,0) B(3,0)	B(1,0) A(3,0)	
199															B(3,0) A(2,1)	
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International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Candidate personal code:

Extended essay - Reflections on planning and progress form

Candidate: *This form is to be completed by the candidate during the course and completion of their EE. This document records reflections on your planning and progress, and the nature of your discussions with your supervisor. You must undertake three formal reflection sessions with your supervisor: The first formal reflection session should focus on your initial ideas and how you plan to undertake your research; the interim reflection session is once a significant amount of your research has been completed, and the final session will be in the form of a viva voce once you have completed and handed in your EE. This document acts as a record in supporting the authenticity of your work. The three reflections combined must amount to no more than 500 words.*

The completion of this form is a mandatory requirement of the EE for first assessment May 2018. It must be submitted together with the completed EE for assessment under Criterion E.

Supervisor: *You must have three reflection sessions with each candidate, one early on in the process, an interim meeting and then the final viva voce. Other check-in sessions are permitted but do not need to be recorded on this sheet. After each reflection session candidates must record their reflections and as the supervisor you must sign and date this form.*

First reflection session

Candidate comments:

I am interested in studying the underlying algorithms within Chopsticks, which is a hand game between two people with complete information. Prior to choosing Chopsticks I was already interested in doing a topic related to combinatorial game theory. I am very competitive in nature, and whenever I engage in any game or competition with a friend I focus most on using logic to find strategies and gain an advantage. My thinking process in these cases is often disorganized, which is why I would like to delve into combinatorial game theory to learn how to structurally analyze and solve a game. Eventually I chose Chopsticks as it was a game my friends and I recently discovered and I had yet to develop strategies for myself. Another reason for choosing Chopsticks is due to it being untouched, as I could not find any extensive research into the game, and I love the idea of exploring something new. It may seem difficult to solve the game without references to other works on the game, though I can refer to investigations on similar deterministic games instead. I also believe the standard game will not be too difficult to solve as it is quite short, which is why I am planning on expanding the rules of the game and re-analyzing it.

Date: May 19, 2018

Supervisor initials:

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Bachillerato Internacional

Interim reflection

Candidate comments:

Finding the winning algorithm was not as simple as I thought. The main problem was that there was no absolute winning strategy, meaning that if both players played perfectly, the game would never end such that the game could not be solved. However, this continuous game only applies if both players play perfectly. Thus, I considered what may be possible winning strategies against opponents that did not know how to play perfectly, which I found and discussed. Expanding the game from here on would only make my research more abstract, which would go against my original purpose of exploring combinatorial game theory and using it solve a game. As such, I instead decided to simplify the game such as to avoid never-ending cycles and allow myself to properly solve the game.

Date: September 18, 2018

Supervisor initials

Final reflection - Viva voce

Candidate comments:

The simplified version of the game was solved and I am satisfied with the results. I solved the game through its game tree, which I chose to do as according to many other solved games the game tree method, although rough, is definitely the most thorough way to solve a game. My only dissatisfaction on this entire investigation is that the standard game could not be solved, and that my research process was roundabout as I started with the standard game and only afterwards analyzed the simplified version. I learned from this that when analyzing something, it is best to start simple and slowly make it complex rather than starting with a complex problem first. I am convinced about this since starting simple allows you to better understand what the problem exactly is and why the solution works, which is difficult to comprehend if you directly jump into the complex version.

Date: December 20, 2018

Supervisor initials